AQA Maths M2

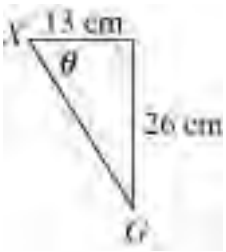
Topic Questions from Papers

Centre of Mass

Answers

1 (a)	$\bar{x} = \dfrac{4\times 90 + 7\times 90}{4+7+8+11}$	M1		moment equation
		A1		correct equation
	$= \dfrac{990}{30} = 33$ cm AG	A1	3	correct distance from correct working
(b)	$\bar{y} = \dfrac{11\times 60 + 7\times 60}{30}$	M1		moment equation
		A1		correct equation
	$= \dfrac{1080}{30} = 36$ cm	A1	3	correct distance
(c)	$\tan\alpha = \dfrac{36}{33}$	M1		use of tan
		A1F		correct expression
	$\alpha = 47.5°$	A1F	3	correct angle follow through $\bar{y}$ from part (b).
	Total		**9**	

(Q2, June 2006)

2 (a)	$(10\times 40)\rho\times 5 + (10\times 60)\rho\times 40$	M1		
	$= (10\times 40 + 10\times 60)\rho\bar{y}$	M1		
		A1		
	$\bar{y} = 26$ cm	A1	4	
(b)	Symmetry of shape	B1	1	
(c)		M1		Attempting subtraction leading to 13 cm
	$\tan\theta = \dfrac{26}{"13"}$	M1		Or inverted, must see 26
		A1		Or inverted
	$\theta = 63°$ (63.4)	A1	4	Accept 117°
	Total		**9**	

(Q4, Jan 2007)

3 (a)	Symmetry of the lamina about PQ	E1	1	Accept 'mirror line'
(b)	Taking moments about AB:			
	$600\rho \times 15 + 100\rho \times 35$	M1A1		Condone lack of ρ
	$= 700\rho\bar{x}$	A1		
	$\bar{x} = 17.857 = 17.9$ cm	A1	4	SC3 17.8
(c)	$\tan\theta = \frac{10}{17.857}$	M1A1		M1 for use of $\tan\theta$
	$= 0.56$			
	Angle is 29.2488…	M1		
	$= 29°$	A1	4	
	Total		**9**	

(Q2, June 2007)

4	$\bar{X} = \frac{25\times1+12\times4+4\times5}{1+4+5}$	M1		Two terms on top correct (+third) and denominator correct
	$= \frac{93}{10}$ or 9.3	A1		
	$\bar{Y} = \frac{10\times1+7\times4+18\times5}{10}$	M1		
	$= \frac{128}{10}$ or 12.8	A1	4	SC3 for interchanged $\bar{X}$ and $\bar{Y}$
	$\therefore$ Centre of mass is at (9.3, 12.8)			
	Total		**4**	

(Q3, June 2008)

5 (a)	Taking moments about AD:			
	$8 \times 10 + 2 \times 15 = 10\,\bar{x}$	M1A1		M1 for moments and 1 term on left correct and 1 term on right
	$\bar{x} = \frac{110}{10}$			
	$= 11$ cm	A1	3	
(b)	5 cm	B1	1	
(c)	(tan) $\theta = \frac{1}{5}$ ie $\frac{(a)-10}{(b)}$	M1		From areas; $\frac{1.4}{5} \Rightarrow \theta = 15.6$ or 15.7
	$= 0.2$	A1ft		
	Angle is $\tan^{-1}(0.2)$	M1		
	$= 11.3°$	A1ft	4	
(d)	Centre of mass is at middle of lamina	E1	1	
	Total		**9**	

(Q4, Jan 2009)

6	$\bar{X} = \frac{3\times15+1\times7+6\times8+10\times12}{3+1+6+10}$ $= \frac{220}{20}$ or 11 $\bar{Y} = \frac{3\times6+1\times14+6\times7+10\times9}{20}$ $= \frac{164}{20}$ or 8.2 $\therefore$ Centre of mass is at (11, 8.2)	M1A1 A1 M1A1 A1	6	M1 for at least 3 multiplication & addition SC 4 (10, 7.4) [omit lamina] ie: B2, B2
	Total		**6**	

(Q2, Jan 2010)

7 (a)	Symmetry	E1	1	Only accept 'symmetry'
(b)	Moments about *B*: $0.4 \times 4 + 0.1 \times 8 = 0.5 \times \bar{x}$ $\bar{x} = \frac{2.4}{0.5}$ = 4.8 cm	M1A1 A1	3	M1 3 terms , 2 correct
	Total		**4**	

(Q3, June 2010)

8 (a)	Moments about line *AD*: $5 \times 30 + 4 \times 10 = 9 \times \bar{x}$ $\bar{x} = \frac{190}{9}$ $= 21.1$ cm	M1A1 A1	 3	M1 2 of 3 terms correct
(b)	Moments about line *AB*: $5 \times 15 + 4 \times 25 = 9 \times \bar{y}$ $\bar{y} = \frac{175}{9}$ $\bar{y} = 19.4$ cm	M1A1 A1	 3	M1 2 of 3 terms correct If moments about *DC*; 10.6 found SC2
(c)	$\tan\theta = \frac{80}{175}$ or $\frac{8.9}{19.4}$ $= 0.4571$ Angle is $\tan^{-1} 0.4571$ $= 24.6°$	M1 A1 A1 A1	 4	M1 use of tan A1 use of 8.9 or 80 $\left(30-(a)\right)$ Or 0.45876 $65.4^\circ \Rightarrow$ M1A1 only
(d)	Moments about the line *PR*: (or *AD* or *BC*) $30m = 4 \times 20$ or $9 \times \frac{80}{9}$ $m = \frac{8}{3}$	M1 A1 A1	 3	
(e)	Centre of mass is at middle of lamina	E1	1	
	Total		**14**	

(Q4, Jan 2011)

9	$\bar{X} = \frac{2\times9+3\times2+8\times3+7\times6}{2+3+8+7}$ $= \frac{90}{20}$ or 4.5 $\bar{Y} = \frac{2\times6+3\times4+8\times8+7\times11}{20}$ $= \frac{165}{20}$ or 8.25 $\therefore$ Centre of mass is at (4.5, 8.25)	M1 A1 M1 A1 A1F	 5	M1: Expression for $\bar{X}$ with no more than one error in the numerator and correct denominator. A1: Correct distance. Accept $\frac{9}{2}$ or $\frac{90}{20}$ or equivalent. M1: Expression for $\bar{Y}$ with no more than one error in the numerator and correct denominator. A1: Correct distance. Accept $\frac{33}{4}$ or $\frac{165}{20}$ or equivalent. A1: Correct coordinates; dependent on M1 M1 Do not accept $\frac{90}{20}$ etc at this stage. SC4: For final answer (8.25, 4.5) award 4 marks. Moments about *B*, (2.5, 4.25) SC2
	Total		**5**	

(Q2, June 2011)

10 (a)(i)	Moments about *AB*: $1.6 \times 4 + 0.4 \times 8 = 2 \times x$ $x = 4.8$ Distance is 4.8 cm	 M1A1 A1	 3	 M1 for 2 terms correct
(ii)	Moments about *AD*: $1.6 \times 6 + 0.4 \times 12 = 2 \times y$ $y = 7.2$ Distance is 7.2 cm	 M1A1 A1	 3	 M1 for 2 terms correct SC2+SC2 for (a)(i) and (a)(ii) reversed
(b)	Moments about *A*: $1.6g \times 6 + 0.4\,g \times 12 = 12 \times T_B$ $T_B = 1.2g = 11.8$ N Resolve vertically: $T_A + T_B = 2g$ $T_A = 0.8g = 7.84$ N	 M1A1 A1 M1 A1	 5	 M1 for 1 side of equation Or using above: moments about *A* $12 \times T_B = 7.2 \times 2g$ (ft for M marks) 1.2 and 0.8 is zero marks If 11.8 and 7.8 as final answer, must lose 1 mark somewhere
	Total		**11**	

(Q3, June 2012)

11 (a)	Symmetry	E1	1	
(b)	Moments about *AB*: $300\sigma.15 + 100\sigma.5 + 300\sigma.15 = 700\sigma.\,x$ $x = \dfrac{9500}{700}$ $= \dfrac{95}{7}$ or 13.6 cm	 M1A1 A1	 3	 (condone lack of σ) M1 needs correct total marks
(c)	Distance from *HG* is 16.4 cm $\tan\theta = \dfrac{15}{16.42857}$ $= 0.913043$ $\theta = 42.3974^\circ$ $\theta = 42^\circ$	B1 M1 A1 A1	 4	 Seeing both 15,16.4 and tan [48° probably B1, M1] NB $\dfrac{13.6}{15}$ etc $\Rightarrow 42^\circ$ no marks
	Total		**8**	

(Q4, Jan 2013)